GCSE Maths – Ratio, Proportion and Rates of Change

General Iterative Processes **(Higher Only)**

Notes

WORKSHEET

General Iterative Processes (Higher Only)

More complex compound growth and decay questions may involve **multiple iterations** (where a process is repeatedly carried out).

There's no set way to solve each question but keeping a clear method and noting down calculations can help to avoid mistakes.

The '**ANS**' button in the calculator can be useful for questions; however, be careful to make the right calculation each time, as it can easily get confusing!

Even when using a calculator, make sure to write down your calculations and results at each step so that the examiner can award marks for workings, even if there is a mistake in the final answer.

Compound Growth and Decay questions

Example: At the start of day 1, Ella has 60 bouquets of flowers in her shop. Each day, 25% of her bouquets are sold. She receives a delivery of 10 fresh bouquets at the end of each day. How many bouquets will she have at the end of day 4?

1. Identify the variables.

$$B = number\ of\ bouquets$$
$$B_0 = initial\ no.\ bouquets$$
$$B_n = no.\ bouquets\ at\ end\ of\ day\ n$$

2. Set up a formula

   Each day the number of bouquets decreases by 25%. *This is equivalent to* $1 - 0.25 = 0.75$ *of the day's starting value. On top of this, we also know that an extra* 10 *bouquets are delivered each day. This can be written as an addition in the formula.*

   $$\boldsymbol{B_{n+1} = 0.75(B_n) + 10}$$

3. Use iterations to find number of bouquets at the end of day 4

   We know $B_0 = 60$. *Using the formula:*

   $B_1 = (0.75 \times 60) + 10 = 55$
   There are 55 bouquets at end of day 1.

   $B_2 = (0.75 \times 55) + 10 = 51.25 = 51\ (nearest\ full\ bouquet)$

   $B_3 = (0.75 \times 51) + 10 = 48.25 = 48\ (nearest\ full\ bouquet)$

   $B_4 = (0.75 \times 48) + 10 = 46$

   At the end of day 4, Ella will have 46 bouquets in her shop.

Example: A plane ticket costs £100 in March. The price increases by 5% at the start of each month over the summer holidays (up to and including September). Does the ticket ever exceed £120? If so, in which month does this happen?

1. Introduce a formula to use for monthly calculations.

$$P_0 = \text{ initial price } = £100$$

$$P_n = \text{ price at the start of month } n$$

$$\boldsymbol{P_{n+1} = P_n \times 1.05}$$

2. Iterate over the summer months.

$$(\text{March}) \ P_0 = £100$$

$$(\text{April}) \ P_1 = 100 \ \times 1.05 = £105$$

$$(\text{May}) \ P_2 = 105 \ \times 1.05 = £110.25$$

$$(\text{June}) \ P_3 = 110.25 \ \times 1.05 = £115.76$$

$$(\text{July}) \ P_4 = 115.76 \ \times 1.05 = £121.55$$

Stop iterating because cost of ticket has exceeded £120.

The price of the ticket rises above £120 in July.

Example: Ben takes out a loan of £1000. After each year, 5% of the outstanding balance is added. Each year Ben will either repay £200 (before the 5% is added) or the full amount left of the balance, whichever is the least. How long will it take Ben to repay the loan?

1. Find out the balance at the end of year 1.

   After 1 year, the balance is £1000 minus the £200 that Ben pays off, plus interest. Balance:
   $$£1000 - £200 + interest = £800 \ + \ interest$$
   $$Interest \ = \ 0.05 \ \times (800) \ = \ £40$$
   $$Balance \ at \ end \ of \ year \ 1 = £800 + £40 = £840$$

2. Continue calculations until balance is less than £200.

$$Balance \ at \ end \ of \ year \ 2 \ = \ 1.05 \ \times \ (840 - 200) = £672$$
$$Balance \ at \ end \ of \ year \ 3 \ = \ 1.05 \ \times (672 - 200) = £495.60$$
$$Balance \ at \ end \ of \ year \ 4 \ = \ 1.05 \ \times (495.6 - 200) = £310.38$$
$$Balance \ at \ end \ of \ year \ 5 \ = \ 1.05 \ \times (310.38 - 200) = £115.90$$

At end of year 5, balance is less than £200.
This means that at the end of year 6, Ben will pay off £115.90 and reduce the balance to 0.
Ben finishes paying off his loan after 6 years, with a final payment of £115.90 at the end of the 6th year.

General Iterative Processes - Practice Questions

1. Tom invested £200 to an international bank account. The bank guarantees a 9% bonus at the end of each year. However, 1.5% bank fees will be deducted from the remaining balance after the bonus has been added. How many years does Tom need to wait to have more than £300 in his bank account?

2. At the start of day 1, James has 50 tyres in his tyre shop. Each day 40% of his tyres are sold. He receives a delivery of 15 new tyres at the end of each day. How many tyres will he have at the end of day 6?

3. Sarah bought a used car worth £500. Every year, the price of the car decreases by 8%. What would be the price of the car after 10 years? Starting from which year will the price of the car drop below £300?

4. Hannah left 25 fish as food for her cat as she left for a vacation for 10 days. The cat needs to eat 5 fish a day to survive. At the start of each day, Hannah's neighbour will feed the cat 2 fish a day. However, after 5 days, Hannah's neighbour stopped feeding the cat as she was out of town. Will the cat have enough portion of fish to last until 10 days? If not, how many days will the cat survive?

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.